Experimental study of the free evolution of the internal gravitational waves affect by Coriolis in a stratified flow

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Abstract

Laboratory experiments are presented to analyze the free evolution of large-scale internal gravitational waves in a stratified water body affected by Coriolis. This investigation was made in a rotating table, where a two-layer stratification was created into a circular basin, using a fresh water layer on top of a dyed salty water layer on the bottom. The control parameters used to characterize the experiments were the dimensionless Burger number, Wedderburn number and aspect ratio of the stratified water column. The free evolution of the internal waves field was measured by tracking the vertical displacements of the pycnocline using an optical method based on digital image analysis. Three sets of experiments were developed for capturing the internal gravity waves behavior as a function of the dimensionless numbers. The results show that the dynamics of the density interface is mainly modulated by the Kelvin-Poincaré interaction and the nonlinear response of solitary type waves, observed in the experiments for specific conditions. Finally, the results show that Earth rotation and nonlinear interactions are important aspects to understand the energy dissipation of internal gravitational basin scale waves.

1 Introduction

Lakes in central-south Chile, as most mid-latitude lakes, are stratified during the summer season. Among the meteorological forcing of the system, wind and heat exchange with the atmosphere play a fundamental role. Such forcing, particularly strong winds, can establish a wide spectrum of responses in the hydrodynamics of a lake (Monismith, 1987), including surface and internal seiches, which can be affected by the Earth rotation, that is, by Coriolis acceleration, when the water body is sufficiently large (Csanady, 1967). In this context, two major classes of wave are possible for a rotating basin; Kelvin or cyclonic waves and Poincaré or anticyclonic waves (Antenucci & Imberger, 2001). The dynamics of these waves may be governed by nonlinear phenomena when the steepening time wave is less than the internal wave period. The steepening of Kelvin wave can degenerate into a solitary-type wave (de la Fuente *et al.*, 2008). These internal waves have great importance on mass transport processes in the lake and on the energy transfer from the wind to the water column. The paper is focused in the behavior of the free evolution of the internal gravitational waves affected by Coriolis in a two-layer stratified flow. A theoretical analysis of governing equations shows that three dimensionless numbers allow us to study the internal wave field. Under the linear theory assumption the initial density interface slope response due to a surface shear stress in a water body is characterized by the Wedderburn number, $W_0 = h_1/2\eta_0$, where h_1 is the thickness of upper layer and η_0 denotes the maximum vertical displacement of the density interface induced by the wind blowing over the lake (Horn *et al.*, 2001). The upwelling conditions are observed about $W_0 = 0.5$. The nonlinear response of the internal waves can be parameterized using the ratio h_1/H_t, where $H_t = h_1 + h_2$ is the total deep of the water column. For $h_1/H_t < 0.5$ nonlinear and non-hydrostatic response of the internal waves are expected. The last dimensionless parameter is the Burger number, $S = R_o/R$, which is the ratio between the Rossby radius of deformation, $R_o = c_i/f$ and a length scale of the water body basin, R, where c_i denotes the internal wave celerity and f is the inertial frequency of the lake. The internal gravitational waves are affected by Coriolis when $S < 1$. The experimental results presented in this paper help to characterize the internal wave field in terms of these three dimensionless numbers.

2 Experimental facility

The experimental study was conducted using a rotating table, whose rotating speed can be varied in the range between 0 and 6 rpm. On top of the rotating table, a circular basin made of transparent plexiglass, with a diameter of 1.8 m and a height of 0.5 m, was mounted. The rotating table is equipped with a frame that can be tilted at different angles then released into the horizontal position in a very short time. Figure 1 show the experimental set-up (Ulloa, 2011). Using this frame, experiments can be conducted by initially tilting the tank, whose angle can be varied from up to 6 to the horizontal axis. The function of this device is to model the wind blowing over a surface lake and the tilt effect of a density interface into the stratified flow, due to the balance force between pressure gradient and surface shear stress. Once reaching the necessary tilt of the tank, for example upwelling conditions, a control quick release system can restore the basin to horizontal position in about 1 second. The quick release of tilting model the sudden absence of wind.
A two-layer stratification was created inside this experimental tank, using a layer of fresh water on top of a layer of dyed salty water, with a given value of the density step $\Delta\rho$. The development of salinity stratification is obtained using a low porosity sponge, with a circular shape, whose diameter is $D_s = 1.8m$ and thickness $e = 8cm$. The stratification method is based in the physical properties that has the sponge: (1) a porous medium of low density; (2) the sponge allows a vertical gravitational flow of water through its area with low velocity.

3 Experimental set

The experiments are focused on analyzing the free evolution of the internal gravity waves developed at the stratified flow affected by the rotation of the system. The main variables to study are the frequency and amplitude of the basin-scale waves in the system. According to a dimensional analysis of the governing equations, these are mainly characterized by three dimensionless parameters: Burger number, Wedderburn number and aspect ratio. Varying these three parameters, different scenarios of thermal vertical structure, maximum amplitudes of internal waves and importance of Coriolis in the water body, are obtained.

To model the experiments data obtained from field campaigns in Lake Villarrica , south

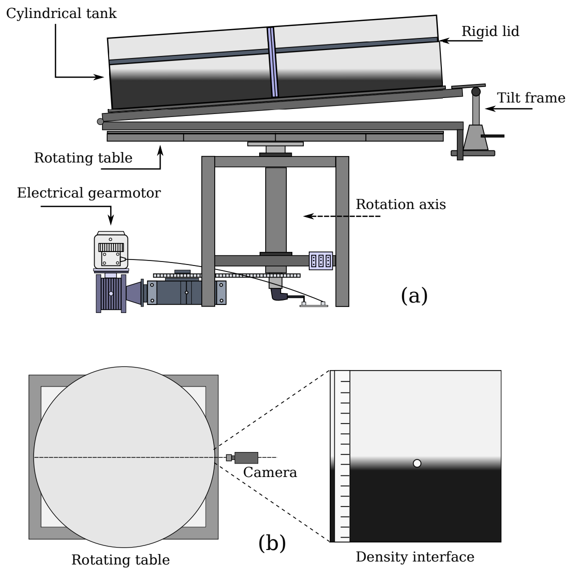

Figure 1: (a) Conceptual model of experimental facilities. (b) Camera placed in the rotating table system.

of Chile (39^oS) (Rozas, 2011) was used. In the summer season, the Burger number is around $S = 0.44$, while the aspect ratio takes values of $h_1/H_t = 0.17$. Moreover, the Wedderburn number fluctuates as a function of wind velocity, being able to observe, under certain synoptic configurations, the so called Puelches's wind (Meruane & Niño, 2005), which reaches high magnitudes. From these results three sets of experiments were defined, which sought to study the response of internal waves as a function of each of the dimensionless parameters in interest:

First experiments series: The first series consisted of 14 experiments, which study the response of the density interface, for the same aspect ratio condition, considering $h_1/H_t \sim 0.2$ and Burger number values in the range of $S \in (0.2-0.6) \cup \infty$. For each value of Burger number two different initial condition for the Wedderbun number, $W = 0.5$ and $W = 1.0$ was considered. This experiment set had, as main goal, homologate the dimensionless number of Lake Villarrica, considering the initial forced condition of the pycnocline.

Second experiments series: The second series consisted of 8 experiments. They had, as main objective, to study the response of the density interface for different aspect ratios, considering the range h_1/H_t in $(0.08 - 0.053)$. Through this set of experiments, the influence of aspect ratio on the nonlinear behavior of internal waves developed in the pycnocline, was analyzed.

Third experiments series: The third series consisted of 4 experiments. They study the response of the density interface for the same aspect ratio condition, $h_1/H_t = 0.5$, considering values of Burger number in the range of $S \in (0.2 - 0.8)$. For each value of the Burger number, a single value of Wedderburn number, $W_0 = 1.0$, was considered. Using this experiment set, the influence of the Coriolis effect in the processes of degeneration and dissipation of the internal gravity waves was analyzed.

4 Experimental results: times series and internal gravitational waves

Figure 2 present a summary of the experimental time series. The plots show the vertical displacement of the pycnocline for experiments of three different sets.

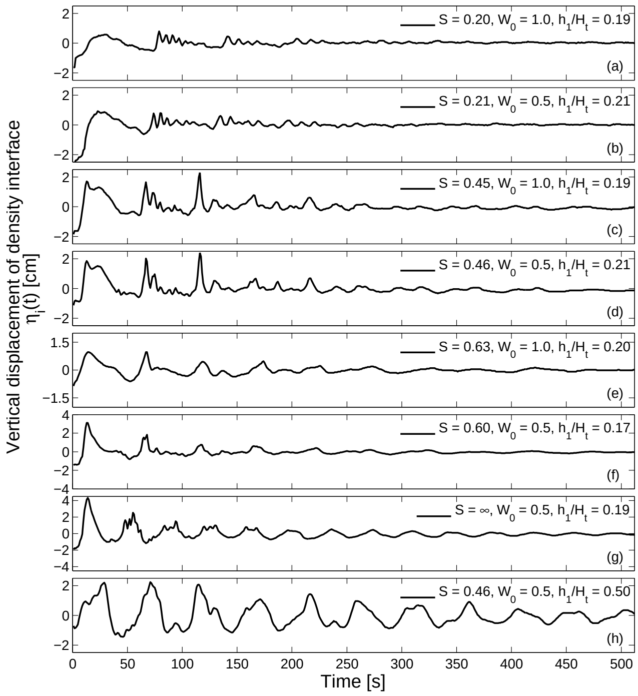

Figure 2: Summary of experimental time series.

To study the field internal gravitational waves the power spectrum density (psd) of experimental time series of vertical displacement of the pycnocline were analyzed (Garcia *et al.*, 2005). This function identify the frequencies that have energy peaks in the inertial range, between inertial rotation frequency, f, and the *Brunt–Väisälä* frequency, $N = \sqrt{-g\rho^{-1}\partial_z\rho}$, where $\partial_z\rho$ denotes the vertical gradient of density water. Each of the energy peaks in the power spectrum density is attributed to a modal frequency of an internal wave. This may correspond to a linear modal response system, to a nonlinear modal interaction between internal waves, or a nonlinear solitary type waves due to the degeneration of basin-scale waves. To identify and classify the internal gravitational waves observed in the power spectrum density the experimental frequencies, which characterize the energy peaks, were compared with the theoretical frequencies obtained of the solution of the eigenvalue problem of the linear wave equation on a stratified flow affected by rotation in a circular basin (Stocker & Imberger, 2003).

5 Classification of internal wave field

5.1 Linear response of internal gravitational waves

For each experiment a classification of the linear field internal waves, as a function of the frequency allocated to the energy peaks in the power spectrum density was obtained. The characterization of the internal wave considered the higher energy peaks, the period of waves, the Burger number of the experiment, the azimuthal and radial internal mode identified, the theoretical and experimental dimensionless frequency of the dominant modes, the sub-inertial or super-inertial characteristic and propagation direction of the internal wave. The results of this classification show that the dominant waves in the system are the fundamental Kelvin wave and the fundamental Poincaré wave. In addition of these waves higher modes of Kelvin wave and Poincaré wave are observed, however, the result show that the Kelvin modes are dominant with respect to the Poincaré modes. The presence of sub-inertial waves is mainly due to the fundamental Kelvin wave, while the super-inertial waves are characterized by the Poincaré waves and the higher Kelvin modes.

5.2 Decomposition of internal waves field and Kelvin-Poincaré Interaction

Using a band pass filter the amount or energy level to the dominant modes, which allowed the reconstitution of the synthetic time series of the vertical displacement of pycnocline for these internal waves, was assigned. Figure 3 shows an example of decomposition result of following modes: ($K_{(1,1)}$, $K_{(2,1)}$, $P_{(1,1)}$,$P_{(2,1)}$). These internal waves were observed in the experiment that homologate the dimensionless number of Lake Villarrica. Is easy to observe the high energy content of Kelvin wave $K_{(1,1)}$. To study the importance of the Kelvin wave $K_{(1,1)}$ and Poincaré wave $P_{(1,1)}$ in the vertical dynamic structure of density interface in time, the interaction between both waves (See Figure 4), considering the synthetic time series obtained from the power spectral density analysis and the decomposition of these modes by using of the band pass filter, was analyzed. Doing a linear superposition of Kelvin mode $K_{(1,1)}$ and Poincaré mode $P_{(1,1)}$ a new time series for the vertical dynamic of the pycnocline was obtained, which was contrasted with the original time series.

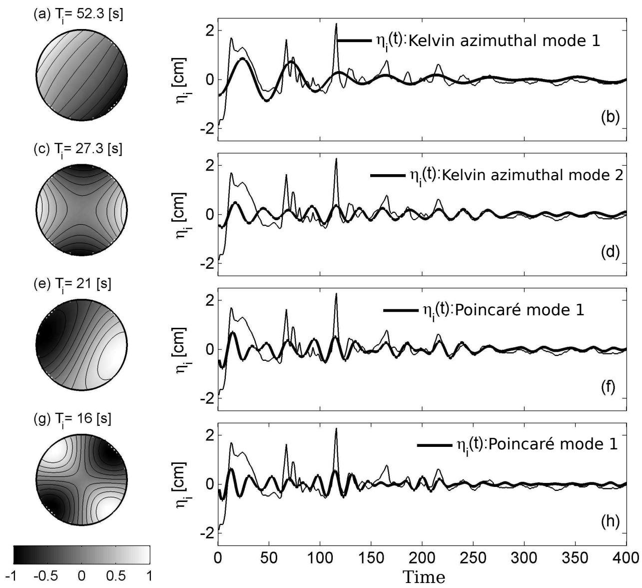

Figure 3: Decomposition of dominant modes of the time series (c) of Figure 2. In all plots the gray line corresponds to the time series of vertical displacement of the pycnoline, while the black line correspond to the time series reconstituted through the use of a bandpass filter, with respect to a particular wave. Panels (a) and (e) correspond to the fundamental modes of the Kelvin and Poincaré waves, respectively. Panels (c) and (d) correspond to the azimuthal mode $K_{(2,1)}$, while Panels (g) and (h) correspond to the azimuthal mode $P_{(2,1)}$.

5.3 Nonlinear response of internal gravitational waves

Theoretical analysis shows that the aspect ratio is the main parameter that characterizes the nonlinear terms of an internal wave. Figure 5 shows the effect of the increasing aspect ratio, between $h_* = 0.1$ and $h_* = 0.5$, on the nonlinearity of waves developed in

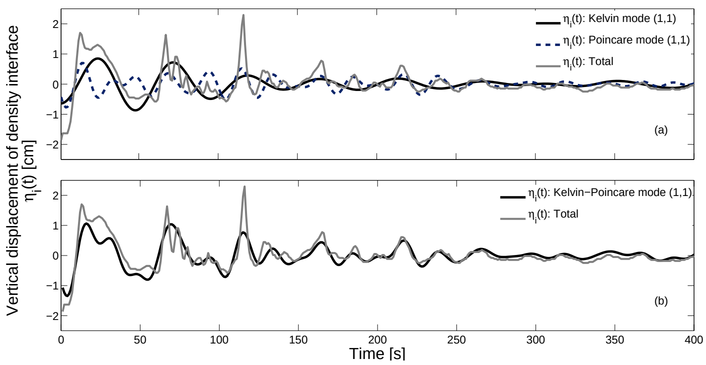

Figure 4: (a) Time series of the density interface of the two main modes, together with the full time series. (b) The linear superposition of these modes is able to characterize the behavior of the density interface. It also explains the presence of the peak observed in the time series, motivated by the overlap of the phases of the waves $K_{(1,1)}$ and $P_{(1.1)}$.

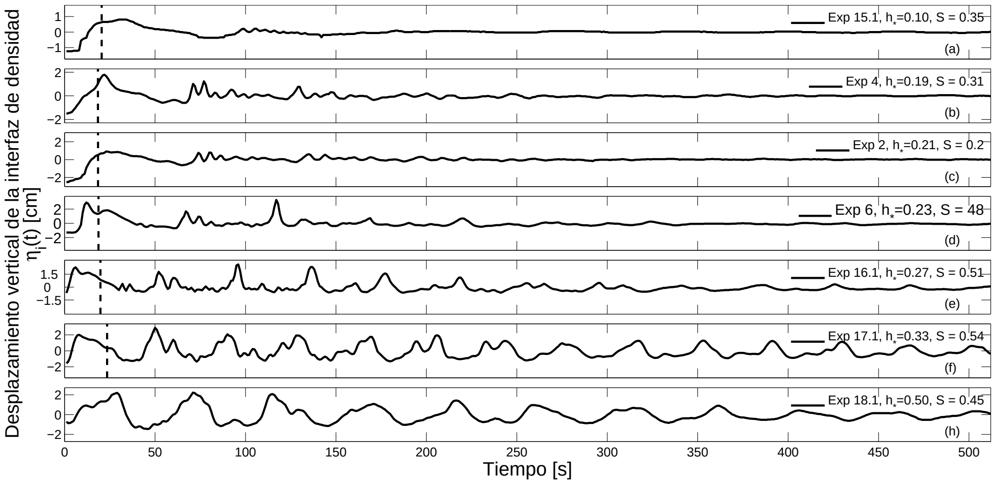

Figure 5: The nonlinear characteristics of internal waves increases as the aspect ratio, h_*, decreases. Likewise, the steepening time decreases as the aspects ratio, h_*, decreases. From Panel (a) through (f), it shows the presence of such waves as solitons, however, for the limiting case of $h_* = 0.5$ is not possible to observe the formation of nonlinear solitary type waves.

the interface density over time. As h_* increases, the number for solitons decays and the steepening of the fundamental wave becomes smaller. Therefore, it is possible to observe the formation of soliton-type waves for $h_* < 0.5$. For $h_* = 0.5$, it is not possible to distinguish the formation of nonlinear waves. All experiments shown in Figure 5 have a Wedderburn number $W_0 = 0.5$ (upwelling condition). Is possible to observe that similar Burger number, as is the case of experiments (d) and (h), and (e) and (f) of Figure 5, Kelvin wave dissipated its energy faster than in experiments with the lower h_*. This result shows that h_* would be an important factor in the mechanism of energy transfer from large-scale waves, such as fundamental Kelvin wave, to small-scale waves such as solitons.

6 Damping mechanics of internal waves

To analyze the scale of energy dissipation, the bulk damping coefficient of the fundamental Kelvin wave, γ_k, observed in each experiment was measured. For this purpose a damped harmonic curve for each time series was fitted, as follows: $\eta(t)/\eta_0 =$

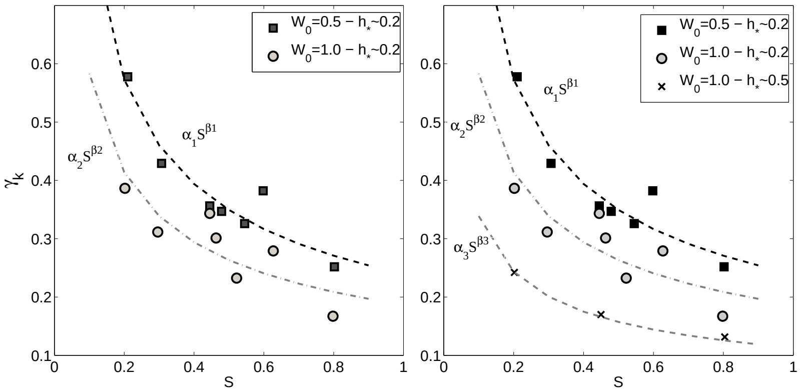

Figure 6: (a) Relationship between the decay coefficient, γ_k, the Wedderburn number, W_0, and the Burger number, S, for an aspect ratio $h_* \sim 0.2$. (b) Comparison of decay coefficient (as function of S) for two different values of aspect ratio, $h_* \sim 0.2$ and 0.5.

$\cos(\omega_k t + \phi)\exp(-\gamma_k t_*)$, where η_0 is the initial amplitude, ω_k is the frequency of the fundamental Kelvin wave, ϕ is the phase of the wave, t_* is the time scaled by the internal wave period, T_i, and γ_k is the decay coefficient. Then, the time scale of experimental decay is characterized as $\tau = T_i/\gamma_k$. Figure 6a shows the dependence of γ_k and S, for an aspect ratio $h_* \in (0.17 - 0.22)$. Both curves show that the rate of decay increases as S decrease, but also show that the rate of decay depends of the initial condition of W_0. Figure 6b shows the influence of nonlinear effects in the mechanism of energy dissipation. For the set of experiments that have an aspect ratio $h_* \sim 0.5$ a significant decrease in the γ_k coefficient was observed.

7 Conclusions

By using a bandpass filter synthetic time series of the main waves observed were obtained, which were fundamental Kelvin and Poincaré modes. This analysis identified the physical and temporal behavior of the fundamental Kelvin wave and particularly the decay coefficient of the wave.

The interaction of Kelvin and Poincaré waves generates the excitation of modes with frequencies that are not attributed to a linear modal base. From this analysis it was found that the coupling phase of the fundamental Kelvin and Poincaré modes are responsible for the major peaks of amplitudes observed in the time series, which shows the presence of significant vertical accelerations, with sufficient inertia. The vertical behavior is a non-hydrostatic wave.

It was possible to identify the presence of nonlinear behavior in internal waves in terms of dimensionless parameters, where the aspect ratio was the main variable that determines the nonlinear characteristics internal waves. Specifically, nonlinear phenomena was observed for $h_1/H_t < 0.5$, while for $h_1/H_t \simeq 0.5$ it was not possible to identify such behavior. The steepening processes and degeneration of the fundamental Kelvin wave into trains of solitons with azimuthal propagation were the main phenomena observed.

The dissipation time scale of a fundamental Kelvin wave took values between 2 and 8 times the period of the Kelvin wave. It was also noted that the scale of decay is strongly influenced by the Coriolis effect and the aspect ratio of the stratification. It was found that the decay coefficient of the fundamental Kelvin wave, γ_k, showed a potential decrease as a function of the Burger number. It demonstrates that the greater influence of rotation on the system, faster is the internal waves decay. From the experiments, it was found that the energy dissipation is minimum for an aspect ratio $h_1/H_t \simeq 0.5$ and it potentially grows to the extent that the aspect ratio decreases. This last result concludes that nonlinear degeneration of internal gravitational waves is an important mechanism to the bulk damping of the internal wave field.

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